STRUCTURAL DESIGN

The Calculation of Steel Frames. J. Heyman

Optimum Design of Beams and Frames in Reinforced Concrete. Ch. Massonnet and M. Save

TIME DEPENDENT BEHAVIOR (CREEP, VISCOELASTICITY)

Influence of Redistribution of Stress on Brittle Creep Rupture of Thick-Walled Tubes Under Internal Pressure. F. K. G. Odquist and J. Erikson

On the Equations of State for Creep. Y. N. Rabotnov

Some Limiting Cases of Non-Newtonian Fluids. H. Ziegler

On Critical States in Viscoelasticity. W. Olszak

On Uniqueness in Linear Viscoelasticity. E. T. Onat and S. Breuer

Thermo-Viscoelastic Stresses in a Sphere with an Ablating Cavity. T. G. Rogers & E. H. Lee

Uniqueness in the Theory of Thermo-Rheologically Simple Ablating Viscoelastic Solids. E. Sternberg and M. E. Gurtin

The volume represents an interesting and valuable collection. The title *Progress in Applied Mechanics* is well chosen.

RICHARD C. ROBERTS

U. S. Naval Ordance Laboratory
White Oak, Silver Spring, Maryland

97[S, X].—R. COURANT & D. HILBERT, *Methods of Mathematical Physics*, volume II by R. COURANT, Interscience Publishers, New York, 1962, xxii + 830 p. Price $17.50.

The two volumes of Courant and Hilbert's *Methoden der mathematischen Physik* have been regarded, since their appearance, as standard source books for applied mathematicians. And this is the second volume of the English version, contributing to "breaking through the language barrier," so to speak.

The preface, by Professor Courant, explains the genesis of the book; this English version is said to have been in preparation ever since the appearance during the last war (1943) of the Interscience Publishers reprint of volume II of the German edition, under license of the United States Government. It also explains the dedication of the book to Kurt Otto Friedrichs as "a natural acknowledgment of a lasting scientific and personal friendship." The polycephalic character of the authorship of the book is also explained (one is reminded here of the skiing picture which was distributed along with many copies of Courant and Friedrichs' book, *Supersonic Flow and Shock Waves*, showing Courant leading a crowd of readily identifiable skiers down a slope, and the resulting shock wave): "The present publication would have been impossible without the sustained unselfish cooperation given to me by friends. Throughout all my career I have had the rare fortune to work with younger people who were successively my students, scientific companions and instructors. Many of them have long since attained high prominence and yet have continued their helpful attitude. Kurt O. Friedrichs and Fritz John, whose scientific association with me began more than thirty years ago, are still actively interested in this work on mathematical physics. . . . To the cooperation of Peter D. Lax and Louis Nirenberg I owe much more than can be expressed by quoting specific details.

Peter Ungar has greatly helped me with productive suggestions and criticisms. Also, Lipman Bers has rendered most valuable help and, moreover, has contributed an important appendix to Chapter IV. . . . Among younger assistants I must particularly mention Donald Ludwig whose active and spontaneous participation has led to a number of significant contributions."

It would be an impossibility to try to mention, even briefly, all the topics which are discussed within the covers of this large volume. Chapter I, entitled Introductory Remarks, describes basic concepts, problems and general lines of approach to their solution. One finds here, in particular, the Cauchy-Kowalewsky existence proof, for analytic solutions of the Cauchy problem, by the method of "majorants." There are two appendices to the chapter, the first on the equation of a minimal surface and the second on the relationship between systems of first-order equations and single differential equations of higher order. Chapter II, bearing the title General Theory of Partial Differential Equations of First Order, centers around the *im kleinen* equivalence of a first-order partial differential equation and a certain system of ordinary differential equations. The Hamilton-Jacobi theory, Hilbert's invariant integral, and contact transformations, are included. There are two appendices to the chapter, the first one on characteristic manifolds, and Haar's uniqueness proof, and the second on the theory of conservation laws, leading to not necessarily smooth, *im grossen* solutions. Chapter III carries the title Differential Equations of Higher Order, and opens with the normal forms for linear and quasi-linear differential operators of second order in two independent variables, followed by a classification of general equations, characteristics. An interesting section contains a lively enumeration and discussion of the chief typical problems of mathematical physics: initial-value problems (Cauchy), boundary-value problems (Dirichlet), mixed problems, Riemann's mapping problem, Plateau's problem for the equation of minimal surfaces, and the jet problem of plane hydrodynamics. It is to be noticed that the formulation of the jet, or Helmholtz problem, a problem whose solution was given by A. Weinstein, is improved over that in the German edition, while an extensive section (in the German edition) on minimal surfaces, a theory to which Courant himself has made outstanding contributions, has been omitted in the present edition. There are two appendices to the chapter, the first on S. L. Sobolev's lemma for estimating a function by means of L_2-bounds of its derivatives, and the second on the uniqueness theorem of Holmgren for analytic equations with arbitrary, not necessarily analytic, Cauchy data. Chapter IV, headed Potential Theory and Elliptic Differential Equations, begins with a rather systematic treatment of potential theory and concludes with a less elementary part, where one finds, among other subjects, Sommerfeld's radiation condition for the reduced wave equation, E. Hopf's maximum principle for elliptic equations, a priori estimates of Schauder, and the solution of elliptic differential equations by means of integral equations (E. E. Levi and D. Hilbert). There is an appendix on boundary-value problems for nonlinear differential equations in several variables, and a supplement (written by L. Bers) on function-theoretic aspects of the theory of elliptic partial differential equations, in particular, the theory of pseudoanalytic functions of L. Bers and I. N. Vekua. This supplement ends with a proof of the Schauder fixed-point theorem. The concluding two chapters are concerned with hyperbolic equations of wave propagation. Chapter V: Hyperbolic Differential

Equations in Two Independent Variables, starts with a review of the basic concept of characteristics, which is then applied to the treatment of the initial-value problems. Among other items, one finds: characteristics and normal forms for hyperbolic systems of first order in two variables; application to the dynamics of compressible fluids; domains of dependence, influence and determinacy; Riemann's method of solution; Cauchy's problem for quasi-linear systems, and for single hyperbolic equations of higher order; and discontinuities of solutions, shocks. There are two appendices to Chapter V, the first is devoted to the application of characteristics as coordinates (in particular, the transition from the hyperbolic to the elliptic case through complex domains, due to H. Lewy, P. Garabedian and H. M. Lieberstein), while the second appendix treats transient problems and the Heaviside operational calculus. Chapter VI: Hyperbolic Differential Equations in More than Two Independent Variables, deals primarily with Cauchy's problem for a single equation of arbitrary order, and with systems of such equations in several unknown functions. The first part of the chapter handles questions of uniqueness, existence, construction, and geometry of solutions, while the second part concentrates on the representation of solutions in terms of the given data, and related questions. In part I one finds: the geometry of characteristics for second- and higher-order operators; applications to hydrodynamics, crystal optics, and magnetohydrodynamics; propagation of discontinuities and Cauchy's problem; oscillatory initial values and asymptotic expansion of the solution; energy integrals and uniqueness for linear symmetric hyperbolic systems and for higher-order equations; and ends with the existence theorem, proved by means of energy inequalities, for symmetric hyperbolic systems. In part II, some of the topics covered are: Cauchy's problem for equations of second order with constant coefficients, the method of spherical means, the method of plane mean values, solution of Cauchy's problem as a linear functional of the data (R. Courant and P. D. Lax), ultrahyperbolic differential equations (Asgeirsson's mean-value theorem), transmission of signals and progressing waves, and Huyghens' principle. It is to be observed that the original chapter, in the German edition, on the classical wave equation in n dimensions, has been reviewed in the present edition, paralleling the recent work of A. Weinstein. The authors retain the original terminology of the German edition in referring to a certain equation as the Darboux equation, while today a great number of mathematicians refer to it as the Euler-Poisson-Darboux equation, in view of the fact that Darboux only considered the one space variable case, whereas Poisson has already considered this equation in three-dimensional space, in his famous investigation of spherical mean values in connection with the wave equation. There is a timely appendix to Chapter VI, dedicated to the theory of ideal functions (S. L. Sobolev) or distributions (L. Schwartz).

It would be easy, as in the case of any book, to mention interesting and important topics which have not been included in the presentation. However, the reader will find plenty to occupy him in this volume. And, as if that were not enough, he can still look forward to the third volume of the series, which is already announced on page one of the present book, as follows: "The present volume, essentially independent of the first, treats the theory of partial differential equations from the point of view of mathematical physics. A shorter third volume will be concerned with existence proofs and with the construction of solutions by finite-difference methods and other procedures."

J. B. DIAZ

University of Maryland
College Park, Maryland

98[V, Z].—K. N. DODD, *Mathematics in Aeronautical Research*, Oxford University Press, New York, 1964, xiii + 130 p., 21 cm. Price $3.40.

Teachers of calculus and elementary differential equations frequently feel the need for fresh, up-to-date applications of the mathematics which they are presenting to the students. Many good textbooks either ignore applications altogether or give a rather bloodless treatment to a succession of stock problems. The present book is designed to supplement these texts by presenting a collection of real problems taken from aeronautical research.

The book was designed for use in the British school systems, but there should be no trouble in using it in first or second year college programs in the United States. The book could also be used by advanced high school students who have had some calculus and a smattering of differential equations.

The book has two strong points: (1) it is directed toward digital computing, and (2) the variety and novelty of the applications is excellent. The chapter titles illustrate these points. They are: 1. Mathematical Concepts, 2. Electronic Computers, 3. Air Composition in an Ascending Fuel Tank, 4. Atmospheric Scattering of a Searchlight Beam, 5. A Computer-Controlled Milling Machine, 6. Accurate Position Determination Using the Gee System, 7. Dynamics of an Ejection Seat Sled, 8. Charge on a Transmission Line, 9. Radiation Doses from Nuclear Attacks, 10. Shattering of Raindrops by Aircraft, 11. More about Raindrops, 12. A Computer Aid to Air Traffic Control, and 13. Supersonic Flow Calculations in Gases.

The reviewer has two criticisms: (1) the treatment of the material, including the introductory chapters, is sometimes overly sketchy even for a book of this type; and (2) there are no references to sources where more information could be found. This latter deficiency limits the use of the book for self-study. A great many teachers should welcome the fresh examples and should find little difficulty in using the book as a supplement to their courses.

RICHARD C. ROBERTS

99[X].—EDWARD OTTO, *Nomography*, Pergamon Press, Ltd., Oxford, England, distributed by The Macmillan Co., New York, 1963, 313 p., 21 cm. Price $10.00.

This book is intended as a text and is not a collection of nomograms. The mathematical level required for an understanding of the subject is quite low (a good high school student should have no difficulty). There are five chapters. Chapter I, as an introduction, deals with analytic geometry and related considerations. Chapters II, III, and IV take up equations with two, three, and many variables, respectively. Chapter V discusses some problems of theoretical nomography.

Y. L. L.